DETERMINISTIC SEASONALITY VERSUS SEASONAL FRACTIONAL INTEGRATION

Luis A. Gil-Alana*
Humboldt Universität zu Berlin, Institut für Statistik und Ökonometrie, Berlin, Germany
University of Navarre, Department of Economics, Pamploma, Spain

ABSTRACT

We make use in this article of a testing procedure suggested by Robinson (1994) for testing deterministic seasonality versus seasonal fractional integration. A new test statistic is developed to simultaneously test both, the order of integration of the seasonal component and the need of seasonal dummy variables. Finite-sample critical values of the tests are computed and, an empirical application, using both, Robinson (1994) and the joint test described in the paper, is also carried out at the end of the article.

JEL Classification: C12; C15; C22
Keywords: Deterministic seasonality; Seasonal fractional integration; Long memory.

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1. Introduction

Modelling the seasonal component of macroeconomic time series has been a major focus of attention in recent years. Deterministic models based on seasonal dummy variables were initially adopted. Later on, however, it was observed that the seasonal component of many series changed over time and stochastic approaches based on seasonal differencing (see eg. Box and Jenkins, 1970) were proposed. In recent years, seasonal-difference models have been extended to allow for other types of long memory behaviour, in particular, allowing seasonal fractional integration. For the purpose of the present paper, we assume that $\{x_t, t = 0, \pm 1, \ldots\}$ is an I(0) process, defined as a covariance stationary process with spectral density function which is positive and finite at any frequency on the interval $(-\pi, \pi]$. We can consider the model

$$(1 - L^4)^d \, y_t = x_t, \qquad t = 1, 2, \ldots \tag{1}$$

where L^4 is the seasonal lag operator ($L^4 y_t = y_{t-4}$) and where d can be any real number. Clearly, if $d = 0$ in (1), $y_t = x_t$, and a weakly autocorrelated process is allowed for. However, for $d > 0$ in (1), y_t is said to be a seasonal long memory process, so-called because of the strong association (in the seasonal structure) between observations widely separated in time. The notion of fractional processes with seasonality was initially suggested by Abrahams and Dempster (1979) and Jonas (1981), and extended in a Bayesian framework by Carlin et. al. (1985) and Carlin and Dempster (1989). Porter-Hudak (1990) applied a seasonally fractionally integrated model like (1) to quarterly US monetary aggregates and other recent empirical applications can be found, for example, in Silvapulle (1995), Ooms (1997) and Gil-Alana and Robinson (2000).

The outline of the paper is as follows: Section 2 describes a version of the tests of Robinson (1994) for testing the order of integration of the seasonal component in raw time series, also including the possibility of seasonal dummy variables in the original series. Section 3 presents a joint test statistic, based on Robinson (1994), for testing simultaneously the order of integration and the need of the seasonal dummy variables. In Section 4 the tests are applied to the consumption and income series of the UK, Canada and Japan while Section 5 contains some concluding comments and extensions.

2. The tests of Robinson (1994) and seasonality

Let's assume that $\{y_t, t = 1, 2, \ldots, T\}$ is the time series we observe and consider the following model,

$$y_t \;=\; \beta_0 \;+\; \sum_{i=1}^{3} \beta_i D_{it} \;+\; x_t \tag{2}$$

$$(1 \;-\; L^4)^d \, x_t \;=\; u_t, \tag{3}$$

where β is a (4x1) vector of unknown parameters; D_1, D_2 and D_3 are seasonal dummy variables and u_t is I(0). Based on (2) and (3), Robinson (1994) proposed a Lagrange Multiplier (LM) test of

$$H_o: \;\; d \;=\; d_o \tag{4}$$

for any real value d_o. Specifically, the test statistic is given by

$$\hat{R} \;=\; \frac{T}{\hat{A}} \frac{\hat{a}^2}{\hat{\sigma}^4} \;=\; \hat{r}^2; \qquad \hat{r} \;=\; \sqrt{\frac{T}{\hat{A}}}\,\frac{\hat{a}}{\hat{\sigma}^2} \tag{5}$$

where

$$\hat{a} \;=\; \frac{-2\pi}{T} \sum_j^{*} \psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \qquad \hat{\sigma}^2 \;=\; \frac{2\pi}{T} \sum_j^{*} g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} \;=\; \frac{2}{T}\left(\sum_j^{*} \psi(\lambda_j)^2 - \sum_j^{*} \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \times \left(\sum_j^{*} \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_j^{*} \hat{\varepsilon}(\lambda_j)\psi(\lambda_j)' \right)$$

$$\psi(\lambda_j) \;=\; \log\left| 2\sin\frac{\lambda_j}{2} \right| + \log\left(2\cos\frac{\lambda_j}{2} \right) \;+\; \log\left| 2\cos\lambda_j \right|; \quad \hat{\varepsilon}(\lambda_j) \;=\; \frac{\partial}{\partial\tau} \log g(\lambda_j;\hat{\tau});$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$ defined as

$$\hat{u}_t = (1-L^4)^{d_o} y_t - \hat{\beta}' w_t; \;\; \hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t' \right)^{-1} \sum_{t=1}^{T} w_t\, (1-L^4)^{d_o} y_t; \quad w_t = (1-L^4)^{d_o} (1, D_1, D_2, D_3)', \text{eva}$$

luated at $\lambda_j = 2\pi j/T$ and g is a known function coming from the spectral density function of

$\hat{u}_t : f(\lambda;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau)$, with $\hat{\tau}$ obtained by minimising $\sigma^2(\tau)$. Note that if u_t is white noise, g $\equiv 1$ and $\hat{A}$ below (5) becomes

$$\frac{2}{T}\sum_j^* \psi(\lambda_j)^2 \qquad (6)$$

which becomes asymptotically $\pi^2/6 \approx 1.645$. Finally, the summation on * in the above expressions are over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1), l = 1, 2, \ldots, s\}$, such that ρ_l, $l = 1, 2, \ldots, s < \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$.

Based on (4), Robinson (1994) showed that under certain regularity conditions,

$$\hat{R} \rightarrow_d \chi_1^2 \qquad as \quad T \rightarrow \infty. \qquad (7)$$

Thus, a 100α%-level test of (4) against H_a: $d \neq d_o$ will reject H_o (4) if $\hat{R} > \chi^2_{1,\alpha}$, where $\Pr ob(\chi_1^2 > \chi^2_{1,\alpha}) = \alpha$. Furthermore, he also showed that the test is efficient in the Pitman sense, i.e. that against local alternatives of form: H_a: $d = d_o + \delta T^{-1/2}$, for $\delta \neq 0$, $\hat{R}$ has a limit distribution given by a $\chi_1^2(\nu)$, with a non-centrality parameter, ν, which is optimal under Gaussianity of u_t. An empirical application of this version of Robinson's (1994) tests is Gil-Alana and Robinson (2000) and Monte Carlo experiments studying its finite-sample behaviour can be found in Gil-Alana (2000a).

Let's suppose now that we want to investigate if the seasonal component of a given time series is deterministic or alternatively, stochastically specified in terms of integrated processes. We can test H_o (4) with $d_o = 0$ in (2) and (3), and the non-rejections of H_o (4) will imply in this case that the seasonal component is deterministic and thus based exclusively on the seasonal dummy variables. On the other hand, testing H_o (4) for values of $d_o > 0$ and imposing $\beta_i = 0$ for i = 1,2 and 3 a priori in (2), the non-rejection values will indicate that the seasonal component is stochastic, either with unit roots (if $d_o = 1$) or with fractional ones (if $d_o \neq 1$). Furthermore, we can also test for seasonal fractional integration incorporating the seasonal dummies in (2), as well as including stationary autoregressions for the seasonal component. In the following section, we present a joint test for testing simultaneously the need of the seasonal dummies and the order of integration of the seasonal component of the series.

3. A joint test of seasonality and the order of integration

Gil-Alana and Robinson (1997) propose a joint test for testing simultaneously the need of a linear time trend and the order of integration in a given time series at the zero frequency. In this section, a similar test is proposed but, instead of looking at the long run or zero frequency, we concentrate on the seasonal component of the series.

We can consider the model given by (2) and (3) and test the null hypothesis:

$$H_o: \; d = d_o \quad and \quad \beta_i = 0 \quad i = 1, 2, 3, \tag{8}$$

against the alternative:

$$H_o: \; d \neq d_o \quad or \quad \beta_i \neq 0 \quad for\ any\ \; i = 1, 2, 3. \tag{9}$$

A joint test is then given by

$$\widetilde{S} = \widetilde{R} + \sum_{t=1}^{T} \widetilde{u}_t w_{2t}' \left(\sum_{t=1}^{T} w_{2t} w_{2t}' - \sum_{t=1}^{T} w_{2t} w_{1t} \times \left(\sum_{t=1}^{T} w_{1t}^2 \right)^{-1} \times \sum_{t=1}^{T} w_{1t} w_{2t}' \right)^{-1} \sum_{t=1}^{T} w_{2t} \widetilde{u}_t \tag{10}$$

where $w_{1t} = (1 - L^4)^{d_o} 1_t$ and $w_{2t} = (1 - L^4)^{d_o} [D_{1t}, D_{2t}, D_{3t}]'$,

$$\widetilde{u}_t = (1 - L^4)^{d_o} y_t - \widetilde{\alpha} w_{1t}; \quad \widetilde{\alpha} = \left(\sum_{t=1}^{T} w_{1t}^2 \right)^{-1} \sum_{t=1}^{T} w_{1t} (1 - L^4)^{d_o} y_{t,}$$

and $\widetilde{R}$ as in (5) but using the $\widetilde{u}_t$ just defined. Then, under H$_o$ (8), $\widetilde{S} \rightarrow_d \chi_2^2$ as T $\rightarrow \infty$, and we would compare (10) with the upper tail of the χ_2^2 distribution. However, we know that in finite samples, the empirical distribution of the tests of Robinson (1994) can vary substantially from the asymptotic results, (see eg. Gil-Alana, 2000b). Thus, we have computed, in Table 1, finite-sample critical values of both statistics, $\hat{R}$ in (5) and $\widetilde{S}$ in (10).

(Table 1 about here)

In both cases we generate Gaussian series obtained by the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986) with 50,000 replications each case, computing $\hat{R}$ in (5) and $\tilde{S}$ in (10) in a model given by (2) and (3). Due to the inclusion of the seasonal dummies in (2), the critical values will be affected by the order of integration in (3), thus we calculate the critical values for d = 0, 0.25, …, (0.25), …, 1.75 and 2, with sample sizes equal to 48, 96 and 120 and nominal sizes of 5% and 1%.

We see that for both statistics, the finite-sample critical values are much higher than those given by the χ^2 distributions, especially if the sample size is small. Thus, when testing the nulls (4) and (8) against the alternatives: H_a: $d \neq d_o$ and (9) with the asymptotic critical values, the tests will reject the null more often than with the finite-sample ones. We went deeper into the examination of these results and observed that the large numbers obtained for the finite-sample critical values were due to the fact that the quantity (6) required in (5) and (10) converges to its asymptotic value (1.645) very slowly. Thus, if T = 48, this quantity is 0.989; if T = 96, it is 1.216; if T = 120, it becomes 1.293 and only approximates 1.645 when T is higher than 300. (eg., 1.624 if T = 360).

We next examine the sizes and the power properties of the tests in finite samples, comparing the results using the finite-sample critical values with those based on the asymptotic results. Tables 2 and 3 report the rejection frequencies of $\hat{R}$ and $\tilde{S}$ first, supposing that there is no need of seasonal dummies (in Table 3) and then including the dummy variables in the regression model (2).

In Table 2, we assume that the true model is given by

$$y_t \;=\; 1 \;+\; x_t; \qquad (1 \;-\; L^4)\, x_t \;=\; \varepsilon_t,$$

with white noise ε_t and look at the rejection frequencies of $\hat{R}$ and $\tilde{S}$ in a model given by (2) and (3) with d_o = 0, 0.25, …, (0.25), …, 1.75 and 2 for a nominal size of 5% and the same sample sizes as in Table 1. Thus, the rejection frequencies corresponding to d_o = 1 will indicate the sizes of the tests and, in case of $\hat{R}$ in (5), the estimated β's should be around 0. We observe that for both test statistics the sizes of the asymptotic tests are too large in all cases, especially for the joint test $\tilde{S}$, though they improve slightly as we increase the number of observations. The higher sizes of the asymptotic tests are also associated with some superior rejection frequencies, being higher the differences when we are close to the null d = 1. Looking at the results with T = 48, we see that the power of $\hat{R}$ is extremely low, especially when using the finite-sample critical values. This is not surprising noting that $\hat{R}$ assumes the inclusion of seasonal dummy variables which are not present in the true model. In that respect, the power of $\tilde{S}$ (which assumes no dummies under the null) is higher, though inferior with the finite-sample values than with the asymptotic results. Increasing the sample size (eg. T = 120) the results of both tests for both types of critical values improve considerably, the rejection probabilities being competitive in all cases when the alternatives are far away from the null.

(Tables 2 and 3 about here)

Table 3 assumes that the true model is given by

$$y_t \;=\; 1 \;+\; D_{1t} \;+\; 2D_{2t} \;+\; 3D_{3t} \;+\; x_t; \quad (1 \;-\; L^4)x_t \;=\; \varepsilon_t,$$

and we perform the same experiment, i.e., computing $\hat{R}$ and $\tilde{S}$ for the same type of alternatives as in Table 2. Thus, the rejection frequencies of $\hat{R}$ with d = 1 will indicate its size while the rejection probabilities of $\hat{R}$ for d ≠ 1 and of $\tilde{S}$ for any d will give us information about the power of the tests. Surprisingly, the results for $\hat{R}$ are practically the same as in Table 2. That means that Robinson's (1994) tests have not much power in relation to the seasonal dummy variables, which makes the joint test statistic $\tilde{S}$ in (10) useful when describing these situations. Looking at the rejection frequencies of $\tilde{S}$, we see that they are very high when using the asymptotic critical values even if the sample size is small. Using the finite-sample ones, they are small for T = 48 if d_o is around 1, however, increasing T, they improve considerably, being higher than 0.900 if $d_o \leq 0.50$ of if $d_o \geq 1.50$ with T = 120.

4. An empirical application

We analyse in this section the quarterly, seasonally unadjusted, consumption and income series for the UK, Canada and Japan. For the UK, the time period is 1955q1-1984.4; for Canada 1960q1-1994q4; and for Japan, 1961q1-1987q4. Consumption is measured as the log of the total real consumption while income is in all cases the log of the total personal disposable income. The series for the UK and Japan were respectively analysed in Hylleberg, Engle, Granger and Yoo (HEGY, 1990) and in Hylleberg, Engle, Granger and Lee (HEGL, 1993) studying its seasonal integrated and cointegrated structure. The same series for the two countries were also examined in Gil-Alana and Robinson (2000), extending the analysis of HEGY (1990) and HEGL (1993) to a fractional model. The latter paper, however, do not consider deterministic dummy variables for describing the seasonal component and thus, this paper improves Gil-Alana and Robinson (2000) in that respect.

Denoting any of the series y_t, we employ throughout the model (2) and (3) with white noise u_t, testing initially H_o (4) for values d_o = 0.00, 0.25, …, (0.25), …, 1.75 and 2.00. Table 4 reports values of the test statistic $\hat{R}$ in (5). We see that for the UK series, H_o (4) cannot be rejected if d_o ranges between 1 and 2, the lowest statistic being achieved in both series at d_o = 1.50. For the Canadian consumption and income, the only non-rejection values appears when d_o = 1.75 and 2, while for Japan, H_o (4) cannot rejected with d_o = 1.75 and 2 for consumption and with d_o = 1.50 and 1.75 for income. On the other hand, we also observe that H_o (4) always results in a rejection for d_o = 0, implying that at least for this simple case of white noise disturbances, the deterministic seasonal models are inappropriate for these series.

(Tables 4 and 5 about here)

In Table 5 we present the statistic $\tilde{S}$ in (10) for the same d_o values as before. We see a few more non-rejection values than in Table 4 and in all cases the non-rejection d_o's in that table form a proper subset of those in Table 5, suggesting that the seasonal dummies were non-necessary when modelling these series. The results in these two tables seem to indicate that a seasonal unit root is present in the UK consumption and income, while for Canada and Japan, higher orders of integration are observed. However, the significance of these results might be due in large part to unaccounted-for I(0) autocorrelated disturbances. Thus, Tables 6 and 7 reports the same statistics as in Tables 4 and 5 but allowing a seasonal autoregressive structure on u_t. We consider a seasonal AR(1) process of form

$$u_t \;=\; \phi\, u_{t-4} \;+\; \varepsilon_t, \tag{11}$$

with white noise ε_t and, though higher order seasonal and non-seasonal AR processes were also performed, the results were very similar to those reported in the tables. A problem here with the estimated AR's coefficients appears in that, though they entail roots that cannot exceed one in absolute value, they can be arbitrarily close to it, thus the disturbances being possibly non-significantly different from a seasonal unit root. In order to solve this problem, we perform Dickey, Hasza and Fuller (DHF, 1984) tests on the residuals of the differenced regression and in those cases where unit roots cannot be rejected, we do not report the statistics and mark with '—' in the tables. We see across Tables 6 and 7 that all of these cases occur when d is a relatively low number and close to 0. This is not surprising if we take into account that (3) with d = 1 is a similar process, (though with very different statistical properties) to (11) with α close to 1. The critical values for the AR(1) case with T = 120 were computed and though we do not report the values here, they were slightly higher than those given in Table 1. Starting with $\hat{R}$ in (5), we see in Table 6 that the non-rejection values occur at exactly the same (d_o/series) combination as in Table 4 with only two extra non-rejected d_o's corresponding to d_o = 1.50 for Canadian and Japanese income.

(Tables 6 and 7 about here)

Similarly for the joint test, in Table 7, the non-rejections also coincide with those in Table 5 for the case of white noise disturbances, again with two extra non-rejected values, this time corresponding to $d_o = 0.75$ for the UK consumption and income series. In view of these results we have further evidence against the need of seasonal dummy variables for all the series in the three countries considered.

As a final remark and following HEGY (1990), HEGL (1993) and Gil-Alana and Robinson (2000), we also investigate if consumption and income may be cointegrated. Using a very simplistic version of the "permanent income hypothesis theory" as discussed for example by Davidson et al. (1978), we can consider a given cointegrating vector (1, -1) and look at the degree of integration of the difference between consumption and income. Thus, in Tables 8 and 9 we again perform $\hat{R}$ and $\tilde{S}$ this time on the differenced series, using both white noise and seasonal AR(1) disturbances. Starting with the case of white noise disturbances (in Table 8) we see that the non-rejection values of $\hat{R}$ take place when d_o ranges between 0.75 and 1.25. Thus, they are smaller by about 0.50-0.75 than those given in Table 4. Similarly, using the joint statistic $\tilde{S}$, the non-rejection values of d_o also are smaller by approximately the same magnitude as before compared with Table 5 and, apart from the case of $d_o = 0.25$ where H_o (8) cannot be rejected now, all the remaining non-rejections occur at the same values of d_o as when using $\hat{R}$. Thus, we find in this table evidence against deterministic seasonality as well as evidence of fractional cointegration at least for this case of white noise disturbances.

(Tables 8 and 9 about here)

Table 9 extends the results of Table 8 to the case of AR(1) disturbances. We see that the non-rejection values of d_o are smaller for both statistics, ranging between 0.50 and 1.50. Comparing the results here with those in Tables 6 and 7, we see that the orders of integration are again smaller for the differenced series, suggesting further evidence in favour of seasonal fractional cointegration.

5. Concluding comments and extensions

A version of the tests of Robinson (1994) for testing the order of integration of the seasonal component of raw time series including seasonal dummy variables has been proposed in this article. Also, a joint test statistic for simultaneously testing the order of integration and the need of the dummies was developed. Both tests have standard null and local limit distributions. However, finite-sample critical values were computed and the values were much higher than those given by the χ^2 distributions. Monte Carlo experiments conducted in the paper showed that the tests based on the asymptotic results have much larger sizes than their corresponding nominal values, these larger sizes being also associated with some superior rejection frequencies compared with the finite-sample-based tests.

The tests were applied to the consumption and income series of the UK, Canada and Japan. The results based on Robinson's (1994) tests indicated that the order of integration of the UK consumption and income fluctuate widely between 1 and 2 while the orders of integration of the Japanese and Canadian series resulted much higher than 1. The joint test statistic was also performed on the series to check if the seasonal dummy variables were in fact required and the results showed that for all of them the deterministic seasonals were inappropriate. Finally, we also performed the tests on the differenced series to check if a seasonally fractionally cointegrated relationship exists between consumption and income. The results showed that the degree of integration of the differenced series was smaller by about 0.50-075 than in the original series, supporting the idea of the permanent income hypothesis.

The results obtained in this article are not directly comparable with those in Gil-Alana and Robinson (2000), the reason being that the latter paper does not include seasonal dummy variables in its regression model. In that respect, we found in this article certain evidence against the deterministic dummies and thus, the conclusions obtained in Gil-Alana and Robinson (2000) remains valid. HEGY (1990) and HEGY (1993) looked respectively at the UK and Japanese series exclusively in terms of seasonally integrated and cointegrated processes and though they allow deterministic seasonality, they do not consider the possibility of seasonal fractional integration. Our results support the idea that the UK consumption and income may both be quarterly I(1) process (as in HEGY, 1990), however, unlike HEGL (1993) we found evidence against this hypothesis for the Japanese case. Finally, and similarly to all these authors, we also found support for the permanent income hypothesis for the three countries considered.

We should also mention that that the test statistics presented in this article have nothing to do with the estimation of the fractional differencing parameter but simply generates computed diagnostics against departures from real values of d. In this context, Ooms (1997) suggests Wald tests based on Robinson's (1994) model, using for the estimation a modified periodogram regression procedure of Hassler (1994), whose distribution is evaluated under simulation. Similar methods based on this and other procedures (eg, Hosoya, 1997) can be applied to these and other macroeconomic time series.

The frequency domain set-up of the tests used in this article may result cumbersome for the practitioners. There also exist time domain versions of Robinson's (1994) tests (cf., Robinson, 1991). However, the preference here for the frequency domain approach is motivated by the somewhat greater elegance it affords especially if the disturbances are weakly autocorrelated. The FORTRAN code used in this application is available from the author upon request.

This article can be extended in several directions. First, the seasonal differenced structure $(1 - L^4)^d$ can be decomposed (as in Gil-Alana and Robinson, 2000) into its long run component $((1 - L)^d)$ and its remaining seasonal components $((1 + L)^d$ and $(1 + L^2)^d)$ and thus, we could test separately each of these components in the presence of seasonal dummies. Also, the joint test statistic developed in the paper can be designed for other versions of the tests of Robinson

(1994), (eg, Gil-Alana, 1999 with monthly integrated structures, and Gil-Alana, 2000c in case of cyclic behaviours). Work in all these directions is now under progress.

References

Abrahams, M. and A.P. Dempster, 1979, Research on seasonal analysis, Progress report of the ASA/Census project on seasonal adjustment, Technical report, Department of Statistics, Harvard University, Cambridge, Massachusetts.

Box, G.E.P. and G.M. Jenkins, 1970, Time series analysis: Forecasting and control, San Francisco, Holden Day.

Carlin, J.B. and A.P. Dempster, 1989, Sensitivity analysis of seasonal adjustments: Empirical cases studies, Journal of the American Statistical Association 84, 6 – 20.

Carlin, J.B., A.P. Dempster and A.B. Jonas, 1985, On methods and moments for Bayesian time series analysis, Journal of Econometrics 30, 67 – 90.

Davidson, J.E., D.F. Hendry, F. Srba and S. Yeo, 1978, Econometric modelling of aggregate time series relationships between consumer's expenditure and income in the UK, Economic Journal 91, 704 – 715.

Dickey, D.A., D.P. Hasza and W.A. Fuller, 1984, Testing for unit roots in seasonal time series, Journal of the American Statistical Association 79, 355 – 367.

Gil-Alana, L.A., 1999, Testing fractional integration with monthly data, Economic Modelling 16, 613 – 629.

Gil-Alana, L.A., 2000a, Modelling seasonality with fractionally integrated processes, Preprint.

Gil-Alana, L.A., 2000b, Evaluation of Robinson's (1994) tests in finite samples, forthcoming in Journal of Statistical Computation and Simulation.

Gil-Alana, L.A., 2000c, Testing stochastic cycles in macroeconomic time series, Preprint.

Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, Journal of Econometrics 80. 241 - 268.

Gil-Alana, L.A. and P.M. Robinson, 2000, Testing seasonal fractional integration in the UK and Japanese consumption and income, forthcoming in Journal of Applied Econometrics.

Hassler, U., 1994, Misspecification of long memory seasonal time series, Journal of Time Series Analysis 15, 19 – 30.

Hosoya, Y., 1997, A limit theory for long-range dependence and statistical inference on related models, Annals of Statistics 25, 105 – 137.

Hylleberg, S., R.F. Engle, C.W.J. Granger and H.S. Lee, 1993, Seasonal cointegration. The Japanese consumption function, Journal of Econometrics 55, 275 – 298.

Hylleberg, S., R.F. Engle, C.W.J. Granger and B.S. Yoo, 1990, Seasonal integration and cointegration, Journal of Econometrics 44, 215 – 238.

Jonas, A.B., 1981, Long memory self similar time series models, Unpublished manuscript, Harvard University, Department of Economics, Harvard.

Ooms, M., 1997, Flexible seasonal long memory and economic time series, Preprint.

Porter-Hudak, S., 1990, An application of the seasonal fractional differenced model to the monetary aggregates, Journal of the American Statistical Association 85, 338 - 344.

Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Wetterling, 1986, Numerical recipes. The art of scientific computing, Cambridge University Press, Cambridge.

Robinson, P.M., 1991, Testing for strong serial correlation and dynamic conditional heteroskedasticity in multiple regression, Journal of Econometrics 47, 67 – 84.

Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420 - 1437.

Silvapulle, P., 1995, A score test of seasonal fractional integration and cointegration, Preprint.

TABLE 1

Finite-sample critical values of $\hat{R}$ in (5) and $\widetilde{S}$ in (10)

Model: $y_t \;=\; \beta_0 \;+\; \sum_{i=1}^{3} \beta_i\, D_{it} \;+\; x_t; \;\; (1 - L^4)^d\, x_t \;=\; \varepsilon_t.$

	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		5%	1%	5%	1%
T = 48	0.00	6.82	9.31	13.91	19.03
	0.25	7.02	9.42	13.88	19.01
	0.50	7.40	9.96	13.81	18.69
	0.75	8.10	10.88	13.37	17.65
	1.00	8.40	11.43	13.10	17.33
	1.25	8.26	11.39	13.16	17.30
	1.50	8.15	11.44	13.14	17.41
	1.75	8.04	11.39	13.33	17.47
	2.00	7.94	11.18	13.42	17.69
	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		5%	1%	5%	1%
T = 96	0.00	5.72	8.20	12.19	16.44
	0.25	5.84	8.37	12.23	16.58
	0.50	6.28	8.90	11.97	16.30
	0.75	6.68	9.38	11.76	15.76
	1.00	6.74	9.33	11.65	15.51
	1.25	6.57	9.13	11.77	15.46
	1.50	6.46	9.05	11.78	15.63
	1.75	6.37	8.96	11.79	15.67
	2.00	6.25	8.93	11.87	15.75
	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		5%	1%	5%	1%
T = 120	0.00	5.37	8.05	11.59	16.35
	0.25	5.50	8.13	11.59	16.11
	0.50	5.95	8.60	11.54	15.53
	0.75	6.36	8.99	11.37	15.40
	1.00	6.23	8.95	11.30	15.35
	1.25	6.03	8.75	11.36	15.26
	1.50	5.92	8.66	11.38	15.42
	1.75	5.87	8.60	11.39	15.72
	2.00	5.77	8.64	11.36	15.71

The critical values of a χ_1^2 distribution are 3.84 at the 5% significance level and 6.63 at the 1% level. For the χ_2^2 distribution are 5.99 and 9.21 respectively.

TABLE 2

Rejection frequencies of $\hat{R}$ and $\widetilde{S}$ in (5) and (10)

True model: $y_t = 1 + x_t; \quad (1 - L^4)^{d_o} x_t = \varepsilon_t; \quad d_o = 1.$

Alternative: $y_t = \beta_0 + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + x_t; \; (1 - L^4)^{d_o} x_t = \varepsilon_t.$

	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 48	0.00	0.297	0.477	0.975	0.996
	0.25	0.089	0.246	0.868	0.967
	0.50	0.020	0.099	0.496	0.797
	0.75	0.044	0.234	0.122	0.514
	1.00	**0.052**	**0.365**	**0.050**	**0.486**
	1.25	0.092	0.534	0.103	0.665
	1.50	0.208	0.745	0.250	0.844
	1.75	0.380	0.879	0.439	0.944
	2.00	0.542	0.954	0.627	0.982
	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 96	0.00	0.959	0.973	0.999	1.000
	0.25	0.869	0.909	0.996	0.999
	0.50	0.367	0.510	0.784	0.941
	0.75	0.037	0.110	0.117	0.406
	1.00	**0.050**	**0.213**	**0.050**	**0.366**
	1.25	0.314	0.678	0.230	0.750
	1.50	0.771	0.957	0.599	0.968
	1.75	0.960	0.997	0.875	0.997
	2.00	0.995	1.000	0.969	0.999
	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 120	0.00	0.991	0.994	1.000	1.000
	0.25	0.961	0.974	0.999	0.999
	0.50	0.602	0.700	0.892	0.974
	0.75	0.068	0.137	0.138	0.432
	1.00	**0.050**	**0.175**	**0.050**	**0.332**
	1.25	0.451	0.741	0.304	0.796
	1.50	0.919	0.986	0.751	0.987
	1.75	0.996	0.999	0.961	1.000
	2.00	0.999	1.000	0.994	1.000

The nominal size is 5% in all cases. The sizes are in bold.

TABLE 3					
Rejection frequencies of $\hat{R}$ and $\widetilde{S}$ in (5) and (10)					
True model: $y_t = 1 + D_{1t} + 2D_{2t} + 3D_{3t} + x_t; \quad (1-L^4)^{d_o} x_t = \varepsilon_t; \quad d_o = 1.$					
Alternative: $y_t = \beta_0 + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + x_t; \; (1-L^4)^{d_o} x_t = \varepsilon_t.$					
	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 48	0.00	0.297	0.477	0.987	0.997
	0.25	0.089	0.246	0.923	0.983
	0.50	0.020	0.099	0.701	0.911
	0.75	0.044	0.234	0.414	0.824
	1.00	**0.050**	**0.365**	0.342	0.838
	1.25	0.092	0.534	0.470	0.908
	1.50	0.208	0.745	0.659	0.962
	1.75	0.380	0.879	0.798	0.987
	2.00	0.542	0.954	0.886	0.996
	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 96	0.00	0.959	0.973	1.000	1.000
	0.25	0.869	0.909	0.996	0.999
	0.50	0.367	0.505	0.877	0.968
	0.75	0.037	0.110	0.421	0.770
	1.00	**0.050**	**0.213**	0.358	0.768
	1.25	0.314	0.678	0.622	0.935
	1.50	0.771	0.957	0.864	0.993
	1.75	0.960	0.997	0.968	0.999
	2.00	0.995	1.000	0.993	1.000
	d_o	$\widetilde{R}$ (H_o: d = d_o)		$\widetilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 120	0.00	0.991	0.994	1.000	1.000
	0.25	0.961	0.974	0.999	0.999
	0.50	0.601	0.700	0.935	0.986
	0.75	0.068	0.137	0.454	0.777
	1.00	**0.050**	**0.175**	0.364	0.749
	1.25	0.451	0.741	0.680	0.947
	1.50	0.919	0.986	0.926	0.997
	1.75	0.996	0.999	0.991	0.999
	2.00	0.999	1.000	0.999	1.000

The nominal size is 5% in all cases. The sizes are in bold.

TABLE 4									
Testing H_o (4) in (2) and (3) with $\hat{R}$ given by (5) with white noise disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	158.32	119.71	74.85	24.69	**2.36'**	**0.10'**	**0.37'**	**1.93'**	**3.90'**
UK Income	152.12	113.33	67.58	18.18	**1.49'**	**0.03'**	**0.37'**	**1.65'**	**3.32'**
CAN Consumption	295.10	243.65	191.62	200.61	182.89	78.61	19.76	**0.24'**	**4.31'**
CAN Income	298.72	246.65	194.32	201.88	157.60	59.61	13.29	**0.15'**	**2.51'**
JAP Consumption	151.97	108.73	62.42	35.52	59.53	36.30	8.63	**0.16'**	**1.32'**
JAP Income	160.05	117.03	72.36	53.83	58.96	21.30	**1.00'**	**2.43'**	8.58

' and in bold: Non-rejection values at the 95% significance level.

TABLE 5									
Testing H_o (8) in (2) and (3) with $\widetilde{S}$ given by (10) with white noise disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	170.49	130.76	73.44	14.68	**0.98'**	**0.01'**	**0.78'**	**2.64'**	**4.72'**
UK Income	188.09	139.06	76.45	14.98	**1.60'**	**0.10'**	**0.37'**	**1.88'**	**3.80'**
CAN Consumption	292.60	242.38	190.38	193.84	154.38	53.28	**8.98'**	**0.36'**	**7.67'**
CAN Income	296.27	245.41	193.02	193.91	128.94	39.81	**5.19'**	**0.69'**	**6.70'**
JAP Consumption	138.95	103.42	59.70	30.42	32.64	14.10	**1.30'**	**0.73'**	**3.83'**
JAP Income	109.45	93.92	59.50	32.94	18.24	**3.04'**	**0.53'**	**4.34'**	**8.00'**

' and in bold: Non-rejection values at the 95% significance level.

TABLE 6									
Testing H_o (4) in (2) and (3) with $\hat{R}$ given by (5) and seasonal AR(1) disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	--	--	13.86	12.35	**1.61’**	**0.76’**	**0.24’**	**0.001’**	**0.15’**
UK Income	--	--	13.49	10.39	**1.70’**	**0.79’**	**0.27’**	**0.01’**	**0.06’**
CAN Consumption	--	--	--	252.13	230.04	54.08	9.41	**4.08’**	**0.23’**
CAN Income	--	--	--	214.48	176.32	33.63	**7.08’**	**2.20’**	**0.01’**
JAP Consumption	--	--	71.22	66.72	55.08	24.34	12.60	**6.37’**	**1.29’**
JAP Income	--	--	--	50.31	45.54	14.19	**7.25’**	**1.18’**	**0.56’**

‘ and in bold: Non-rejection values at the 95% significance level. ‘—‘ means that the disturbances contains a seasonal unit root.

TABLE 7									
Testing H_o (8) in (2) and (3) with $\widetilde{S}$ given by (10) and seasonal AR(1) disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	--	--	12.25	**9.18’**	**0.89’**	**0.42’**	**0.09’**	**0.04’**	**0.36’**
UK Income	--	--	12.90	**9.25’**	**1.32’**	**0.60’**	**0.18’**	**0.008’**	**0.11’**
CAN Consumption	--	--	--	201.39	178.07	34.30	**6.55’**	**1.94’**	**0.14’**
CAN Income	--	--	--	200.22	134.75	24.11	**7.22’**	**2.14’**	**0.07’**
JAP Consumption	--	--	50.76	42.29	28.50	12.95	**7.07’**	**2.09’**	**0.09’**
JAP Income	--	--	--	21.44	13.62	**6.77’**	**2.84’**	**0.26’**	**1.29’**

‘ and in bold: Non-rejection values at the 95% significance level. ‘—‘ means that the disturbances contains a seasonal unit root.

TABLE 8									
Testing H_o (4) in (2) and (3) with $\hat{R}$ given by (5) with white noise disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK: C_t - Y_t	134.91	63.08	9.37	**0.02'**	**1.75'**	**4.41'**	7.21	9.78	12.04
CANADA: C_t - Y_t	111.10	61.85	12.65	**0.01'**	**3.76'**	10.82	16.61	20.56	23.32
JAPAN: C_t - Y_t	108.06	69.22	23.84	**2.92'**	**0.32'**	**4.13'**	8.22	11.15	13.16
Testing H_o (8) in (2) and (3) with $\tilde{S}$ given by (10) with white noise disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK: C_t - Y_t	53.72	38.10	**5.93'**	**0.19'**	**2.39'**	**5.21'**	**8.00'**	12.45	12.56
CANADA: C_t - Y_t	21.74	19.05	**7.69'**	**0.26'**	**8.05'**	17.90	25.19	30.16	33.63
JAPAN: C_t - Y_t	25.93	12.86	**2.26'**	**1.97'**	**0.09'**	**2.57'**	**5.87'**	18.50	20.41

' and in bold: Non-rejection values at the 95% significance level.

TABLE 9									
Testing H_o (4) in (2) and (3) with $\hat{R}$ given by (5) with AR(1) disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK: C_t - Y_t	--	--	**1.14'**	**0.10'**	**0.82'**	**1.63'**	**2.55'**	7.47	9.40
CANADA: C_t - Y_t	--	--	**0.11'**	**0.36'**	**2.42'**	**5.22'**	8.64	11.95	14.77
JAPAN: C_t - Y_t	--	--	**0.56'**	**1.46'**	**0.77'**	**0.04'**	**1.56'**	7.95	9.09
Testing H_o (8) in (2) and (3) with $\tilde{S}$ given by (10) with AR(1) disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	10.75	20.00
UK: C_t - Y_t	--	10.01	**0.15'**	**0.42'**	**1.30'**	**2.16'**	**8.09'**	13.96	16.81
CANADA: C_t - Y_t	--	12.28	**6.77'**	**3.58'**	**3.91'**	**6.74'**	**10.58'**	14.46	18.08
JAPAN: C_t - Y_t	--	12.95	**1.92'**	**0.11'**	**0.46'**	**0.04'**	**8.80'**	12.35	14.00

' and in bold: Non-rejection values at the 95% significance level. '—' means that the disturbances contains a seasonal unit root.